International Journal of Research in Advent Technology, Vol.6, No.10, October 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

k-Square Difference Mean Labeling of $K_{1,n}$ Related Graphs

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Abstract-Square sum labeling was introduced by K. Ajitha, S. Arumugam and K.A. Germina. Square difference labeling was introduced by J. Shiama. Mean labeling was introduced by S. Somasundaram and R. Ponraj. Further, it was studied by B. Gayathri and R. Gopi. We had extended this notion to a labeling called k-square difference mean labeling of graphs. In this paper, we investigate the k-square difference mean labeling for some special graphs and investigate k-square difference mean labeling of the graph $spl(K_{1,n})$, the graph $P_n@ K_{1,m}$, the graph $C_n@K_{1,m}$, the graph $D_2(K_{1,n})$, and the graph $2C_n@ K_{1,m}$.

Keywords: k-square difference mean labeling (k-SDML), k-square difference mean graph (k-SDMG).
AMS Subject Classification: *05C78*

1. INTRODUCTION

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sence of Frank Harary[2]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and the edge set of a graph G.

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling). For an excellent survey on graph labeling, we refer to Gallian [6].

Square sum labeling was introduced by K. Ajitha, S. Arumugam and K.A. Germina[1]. Square difference labeling was introduced by J. Shiama[7]. Mean labeling was introduced by S. Somasundaram and R. Ponraj[8]. k-square difference mean labeling of graphs was introduced by B. Gayathri and R. Thayalarajan[3]. k-square difference mean labeling of some cycle related graphs was introduced by B. Gayathri and R. Thayalarajan[4]. k-square difference mean labeling of some special graphs was introduced by B. Gayathri and R. Thayalarajan[5].

1.1. Definition

A graph $G = (p, q)$is said to have a ***k*-square difference mean labeling (*k*-SDML)** if there exists a bijection$f: V(G) \to \{k-1, k, k+1, \ldots, k+(p-2)\}$ such that the induced function $f^*: E(G) \to N$ given by $f^*(uv) = \left\lceil \frac{|f(u)^2 - f(v)^2|}{2} \right\rceil$ or $\left\lfloor \frac{|f(u)^2 - f(v)^2|}{2} \right\rfloor$ for every edge $uv \in E(G)$ are all distinct.

A graph G is said to be a ***k*-square difference mean graph (*k*-SDMG)** if it admits a k-square difference mean labeling, where k is any positive integer greater than or equal to 1.

1.2. Definition

For a graph G, the split graph is obtained by adding to each vertex v, a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G. The resultant graph is denoted as $spl(G)$.

1.3. Definition

Sparklers is a graph obtained by joining an end vertex of a path P_n to the centre of a star $K_{1,m}$ and it is denoted by $P_n@K_{1,m}$.

1.4. Definition

The shadow graph $D_2(G)$ of a connected graph G is obtained by taking two copies of G, say G', G'' and joining each vertex u' of G' to the neighbours of corresponding vertex v' of G''.

1.5. Definition

The Butterfly graph is obtained by two even cycles of the same order sharing a common vertex with an arbitrary number of pendant edges attached at the common vertex.

2. MAIN RESULTS

Theorem 2.1. The split graph $spl(K_{1,n})$ is a k-square difference mean graph for all $k \geq 1$ and for all $n \geq 2$.

Proof: Let the vertices of $spl(K_{1,n})$ be $\{u_1, u_2, \dots, u_n\} \cup \{u'_1, u'_2, \dots, u'_n\} \cup \{v_1, v_2\}$ and let the edges of $spl(K_{1,n})$ be $\{e_1, e_2, \dots, e_n\} \cup \{e'_1, e'_2, \dots, e'_n\} \cup \{g_1, g_2, \dots, g_n\}$, which are denoted as in Fig. 1.

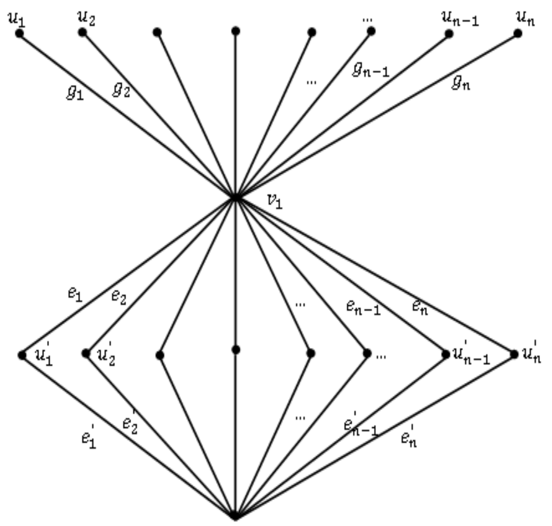

Fig. 1: *Ordinary labeling of* $spl(K_{1,n})$

We know that $|V(spl(K_{1,n}))| = 2n + 2$ and $|E(spl(K_{1,n}))| = 3n$.
First we label the vertices as follows:
Define $f: V(spl(K_{1,n})) \to \{k-1, k, k+1, k+2, \dots, k+(p-2)\}$ by

$f(u_i) = k + n + i;\qquad 1 \leq i \leq n$
$f(u'_i) = k + i;\qquad 1 \leq i \leq n$
$f(v_i) = k + i - 2;\qquad 1 \leq i \leq 2$

Then, the induced edge labels of $spl(K_{1,n})$ are
For $1 \leq i \leq n$,

$$f^*(e_i) = \begin{cases} \frac{1}{2}(i^2 + 2k(i+1) - 1); & i \text{ is odd} \\ \frac{1}{2}(i^2 + 2k(i+1)); & i \text{ is even} \end{cases}$$

For $1 \leq i \leq n$,

$$f^*(e'_i) = \begin{cases} \frac{1}{2}(i^2 + 2ki - 1); & i \text{ is odd} \\ \frac{1}{2}(i^2 + 2ki); & i \text{ is even} \end{cases}$$

If n even, $1 \leq i \leq n$

$$f^*(g_i) = \begin{cases} \frac{1}{2}(n^2 + i(i + 2n + 2k) + 2kn + 2k - 1); & i \text{ is odd} \\ \frac{1}{2}(n^2 + i(i + 2n + 2k) + 2kn + 2k - 2); & i \text{ is even} \end{cases}$$

If n odd, $1 \leq i \leq n$

$$f^*(g_i) = \begin{cases} \frac{1}{2}(n^2 + i(i + 2n + 2k) + 2kn + 2k - 2); & i \text{ is odd} \\ \frac{1}{2}(n^2 + i(i + 2n + 2k) + 2kn + 2k - 1); & i \text{ is even} \end{cases}$$

Clearly, the induced edge labels are distinct. Hence, the split graph $spl(K_{1,n})$ is a k-square difference mean graph for all $k \geq 1$ and for all $n \geq 2$.

Illustration 2.2. 8-SDML of $spl(K_{1,11})$ is shown in Fig. 2.

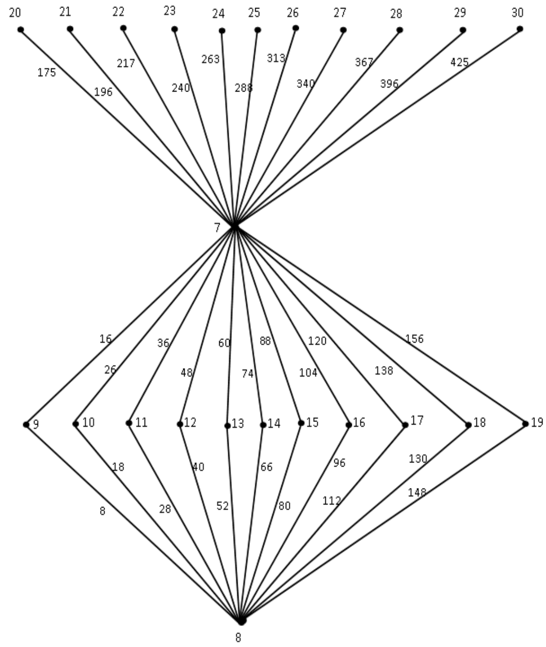

Fig. 2: 8-*SDML of* $spl(K_{1,11})$

Illustration 2.3. 3-SDML of $spl(K_{1,8})$ is shown in Fig. 3.

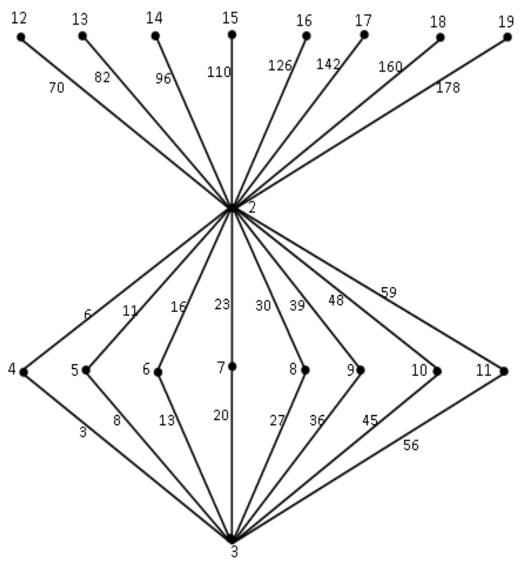

Fig. 3: 3-*SDML of* $spl(K_{1,8})$

Theorem 2.4. The sparklers $P_n@K_{1,m}$ is a k-square difference mean graph for all $k \geq 1$ and for all $n, m \geq 2$.

Proof: Let the vertices of $P_n@K_{1,m}$ be $\{u_1, u_2, \dots, u_n\} \cup \{v_1, v_2, \dots, v_m\}$ and let the edges of $P_n@K_{1,m}$ be

$\{e_1, e_2, \dots, e_{n-1}\} \cup \{e_1', e_2', \dots, e_m'\}$, which are denoted as in Fig. 4.

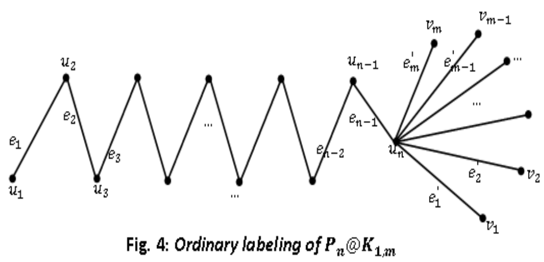

Fig. 4: *Ordinary labeling of $P_n@K_{1,m}$*

We know that $|V(P_n@K_{1,m})| = n+m$ and $|E(P_n@K_{1,m})| = n+m-1$.

First we label the vertices as follows:

Define $f: V(P_n@K_{1,m}) \to \{k-1, k, k+1, k+2, \dots, k+(p-2)\}$ by

$$f(u_i) = k+i-2; \qquad 1 \le i \le n$$

$$f(v_i) = k+n+i-2; \quad 1 \le i \le m$$

Then, the induced edge labels of $P_n@K_{1,m}$ are

$$f^*(e_i) = k+i-1; \qquad 1 \le i \le n-1$$

For $1 \le i \le m$,

$$f^*(e_i') = \begin{cases} k+n-1; & i = 1 \\ 2(k+n-1); & i = 2 \\ \frac{1}{2}(i^2 + 2i(n-1) + 2i(k-1) - 1); & i \text{ is odd}, i \ne 1 \\ \frac{1}{2}\left(i^2 + 2i(n-1) + 2i(k-1)\right); & i \text{ is even}, i \ne 2 \end{cases}$$

Clearly, the induced edge labels are distinct. Hence, the sparklers $P_n@K_{1,m}$ is a k-square difference mean graph for all $k \ge 1$ and for all $n, m \ge 2$.

Illustration 2.5. 3-SDML of $P_{17}@K_{1,9}$ is shown in Fig. 5.

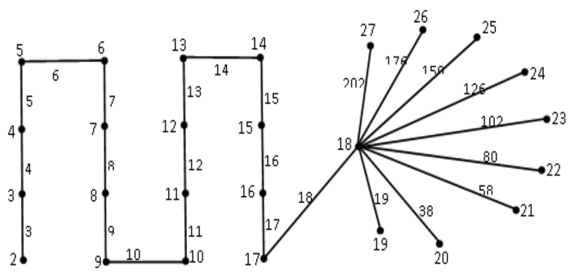

Fig. 5: 3-SDML *of* $P_{17}@K_{1,9}$

llustration 2.6. 8-SDML of $P_{11}@K_{1,6}$ is shown in Fig. 6.

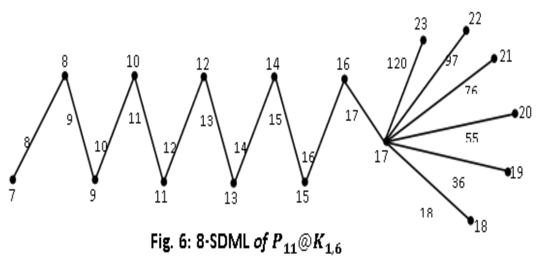

Fig. 6: 8-SDML *of* $P_{11}@K_{1,6}$

Theorem 2.7. The one-point union graph $C_n@K_{1,m}$ is a k-square difference mean graph for all $k \ge 1$, for all $n \ge 4$ and for all $m \ge 2$.

Proof: Let the vertices of $C_n@K_{1,m}$ be $\{u_1, u_2, \dots, u_n\} \cup \{v_1, v_2, \dots, v_m\}$ and let the edges of $C_n@K_{1,m}$ be $\{e_1, e_2, \dots, e_n\} \cup \{e_1', e_2', \dots, e_m'\}$, which are denoted as in Fig. 7.

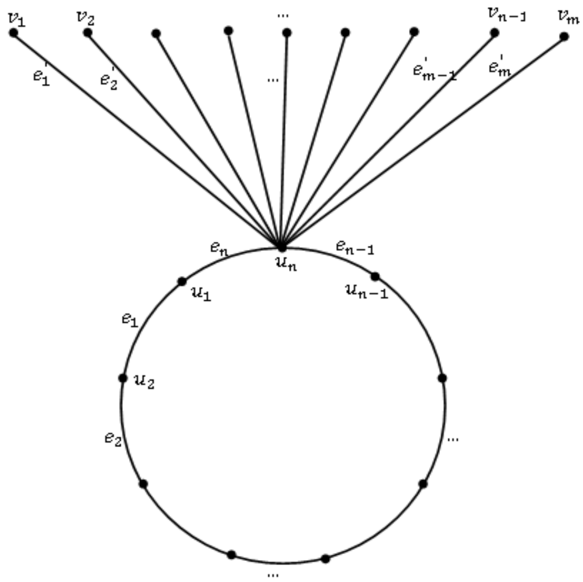

Fig. 7: *Ordinary labeling of $C_n@K_{1,m}$*

We know that $|V(C_n@K_{1,m})| = n+m$ and $|E(C_n@K_{1,m})| = n+m$.

First we label the vertices as follows:

Define $f: V(C_n@K_{1,m}) \to \{k-1, k, k+1, k+2, \dots, k+(p-2)\}$ by

$$f(u_i) = k+i-2; \qquad 1 \le i \le n$$

$$f(v_i) = k+n+i-2; \qquad 1 \le i \le m$$

Then, the induced edge labels of $C_n@K_{1,m}$ are

For $1 \le i \le n$,

$$f^*(e_i) = k+i-1; \qquad 1 \le i \le n-1$$

$$f^*(e_n) = \begin{cases} \frac{1}{2}(n(n+2k-4) - 2k + 4); & n \text{ is even} \\ \frac{1}{2}(n(n+2k-4) - 2k + 3); & n \text{ is odd} \end{cases}$$

For $1 \le i \le m$,

$$f^*(e_i') = \begin{cases} n+k-1; & i = 1 \\ \frac{i}{2}\left((i+2k+2n-4)\right) - 1; & i \text{ is odd},\ i \ne 1 \\ \frac{i}{2}\ (i+2k+2n-4); & i \text{ is even} \end{cases}$$

Clearly, the induced edge labels are distinct. Hence, the one-point union graph $C_n@K_{1,m}$ is a k-square difference mean graph for all $k \ge 1$, for all $n \ge 4$ and for all $m \ge 2$.

Illustration 2.8. 8-SDML of $C_9@K_{1,11}$ is shown in Fig. 8.

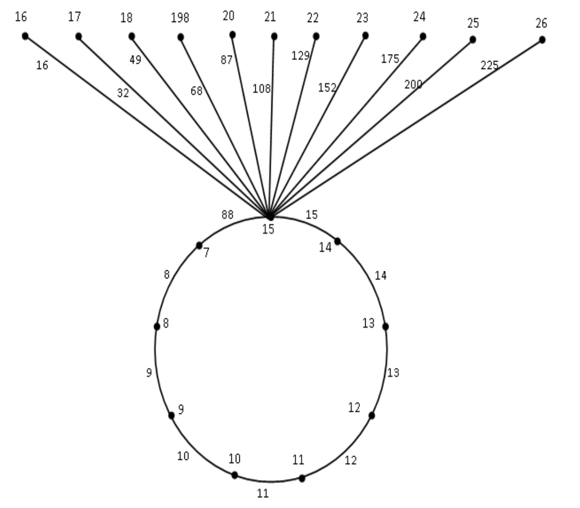

Fig. 8: 8-*SDML of* $C_9@K_{1,11}$

Illustration 2.9. 5-SDML of $C_{14}@K_{1,9}$ is shown in Fig. 9.

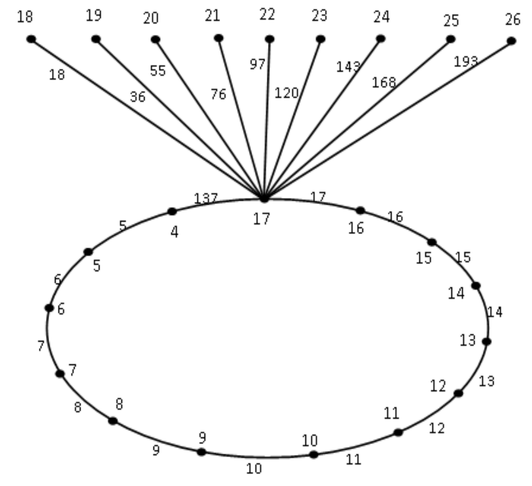

Fig. 9: 5-*SDML of* $C_{14}@K_{1,9}$

Theorem 2.10. The shadow graph $D_2(K_{1,n})$ is a k-square difference mean graph for all $k \geq 1$ and for all $n \geq 2$.

Proof: Let the vertices of $D_2(K_{1,n})$ be $\{u_1, u_2, \dots, u_n\} \cup \{u_1', u_2', \dots, u_n'\} \cup \{v_1, v_2\}$ and let the edges of $D_2(K_{1,n})$ be $\{e_1, e_2, \dots, e_n\} \cup \{e_1', e_2', \dots, e_n'\} \cup \{a_1, a_2, \dots, a_n\} \cup \{a_1', a_2', \dots, a_n'\}$, which are denoted as in Fig. 10.

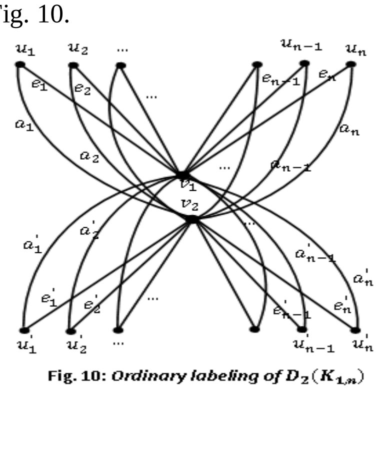

Fig. 10: *Ordinary labeling of* $D_2(K_{1,n})$

We know tha $D_2(K_{1,n}))\big| = 2n + 2$ and $\big|E(D_2(K_{1,n}))\big| = 4$...

First we label the vertices as follows

Define $f: V\big(D_2(K_{1,n})\big) \to \{k-1, k, k+1, k+2, \dots, k+(p-2)\}$ by

$$f(u_i) = k + 2i - 1; \qquad 1 \leq i \leq n$$
$$f(u_i') = k + 2i; \qquad 1 \leq i \leq n$$
$$f(v_i) = k + i - 2; \qquad 1 \leq i \leq 2$$

Then, the induced edge labels of $D_2(K_{1,n})$ are as follows

$$f^*(e_i) = 2i^2 + 2ki - 2i; \quad 1 \leq i \leq n$$
$$f^*(e_i') = 2i^2 + 2ki; \qquad 1 \leq i \leq n$$
$$f^*(a_i) = 2i(i + k - 1) - k; \quad 1 \leq i \leq n$$
$$f^*(a_i') = 2i(i + k) + k; \qquad 1 \leq i \leq n$$

Clearly, the induced edge labels are distinct. Hence, the shadow graph $D_2(K_{1,n})$ is a k-square difference mean graph for all $k \geq 1$ and for all $n \geq 2$.

Illustration 2.11. 6-SDML of $D_2(K_{1,9})$ is shown in Fig. 11.

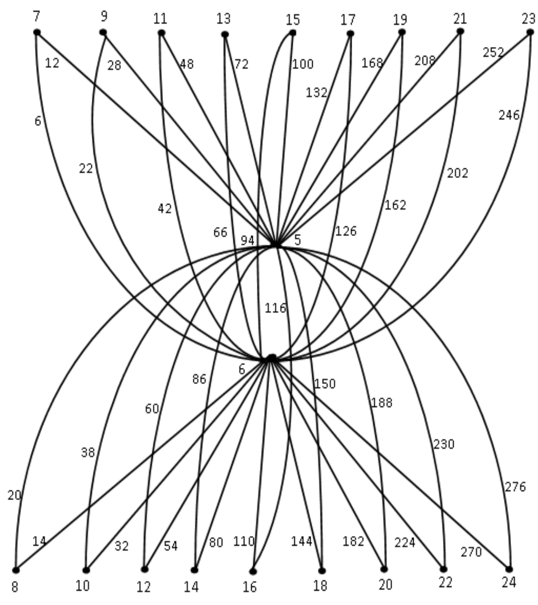

Fig. 11: 6-*SDML of* $D_2(K_{1,9})$

Illustration 2.12. 3-SDML of $D_2(K_{1,6})$ is shown in Fig. 12.

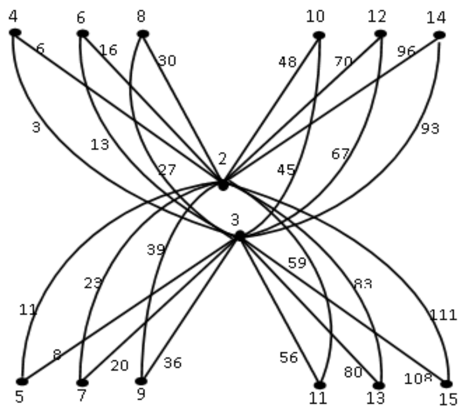

Fig. 12: 3-*SDML of* $D_2(K_{1,6})$

Theorem 2.13. The butterfly graph $2C_n@K_{1,m}, (n \geq 4,\ m \geq 2)$ is a k-square difference mean graph for all $k \geq 1$.

Proof: Let the vertices of the butterfly graph $2C_n@K_{1,m}$ be $\{u_1, u_2, \ldots, u_n\} \cup \{u'_1, u'_2, \ldots, u'_{n-1}\} \cup \{v_1, v_2, \ldots, v_m\}$ and let the edges of the butterfly graph $2C_n@K_{1,m}$ be $\{e_1, e_2, \ldots, e_n\} \cup \{e'_1, e'_2, \ldots, e'_n\} \cup \{e''_1, e''_2, \ldots, e''_m\}$, which are denoted as in Fig. 13.

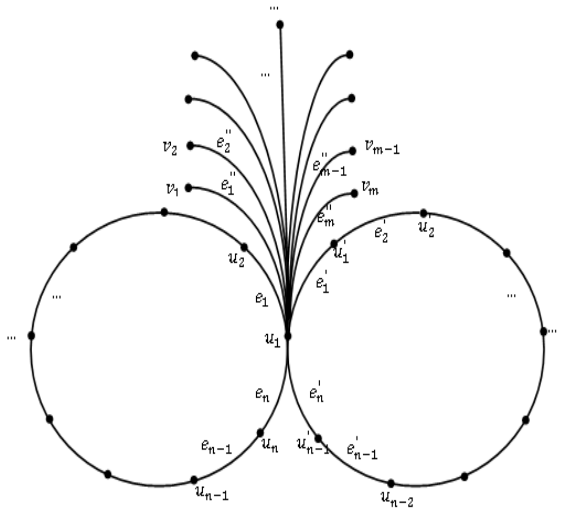

Fig. 13: *Ordinary labeling of* $2C_n@K_{1,m}$

We know that $\left|V(2C_n@K_{1,m})\right| = 2n + m - 1$ and $\left|E(2C_n@K_{1,m})\right| = 2n + m$.

First we label the vertices as follows:

Define $f: V\left(2C_n@K_{1,m}\right) \rightarrow \{k-1, k, k+1, k+2, \ldots, k+(p-2)\}$ by

$$f(u_i) = \begin{cases} k-1; & i = 1 \\ k + 2(i-2); & 2 \leq i \leq n \end{cases}$$

$$f(u'_i) = k + 2(i-2) + 3; \quad 1 \leq i \leq n-1$$

$$f(v_i) = k + 2n + i - 3; \quad 1 \leq i \leq m$$

Then, the induced edge labels of $2C_n@K_{1,m}$ are

For $1 \leq i \leq n$,

$$f^*(e_i) = \begin{cases} k; & i = 1 \\ 2k + 4i - 6; & 2 \leq i \leq n-1 \\ (k-1)(2n-3) + 2n^2 - 6n + 4; & i = n \end{cases}$$

For $1 \leq i \leq n$,

$$f^*(e'_i) = \begin{cases} 2k + 4i - 4; & 1 \leq i \leq n-1 \\ (k-i)(2n-2) + 2n^2 - 4n + 2; & i = n \end{cases}$$

For $1 \leq i \leq m$,

$$f^*(e''_i) = \begin{cases} \frac{1}{2}\left(i(i + 4n + 2k - 6) + 4n(n + k - 3) - 4(k-2) - 1\right); & i \text{ is odd} \\ \frac{1}{2}\left(i(i + 4n + 2k - 6) + 4n(n + k - 3) - 4(k-2)\right); & i \text{ is even} \end{cases}$$

Clearly, the induced edge labels are distinct. Hence, the butterfly graph $2C_n@K_{1,m}, (n \geq 4,\ m \geq 2)$ is a k-square difference mean graph for all $k \geq 1$.

Illustration 2.14. 5-SDML of $2C_9@K_{1,9}$ is shown in Fig. 14.

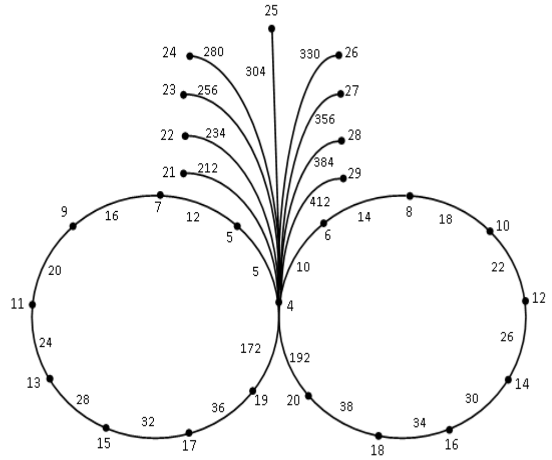

Fig. 14: 5-SDML *of* $2C_9@K_{1,9}$

Illustration 2.15. 18-SDML of $2C_{11}@K_{1,14}$ is shown in Fig. 15.

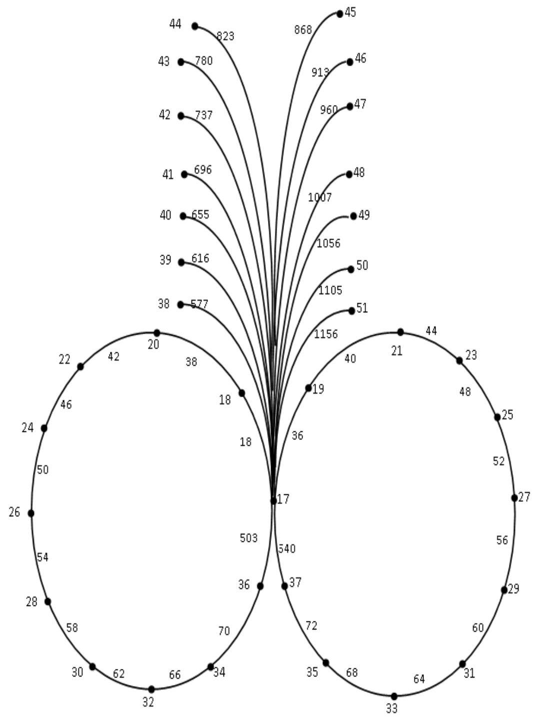

Fig. 15: 18-SDML *of* $2C_{11}@K_{1,14}$

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